① Modular Ratio

$$P_s = \frac{P \cdot E_s}{E_c + E_s} = \frac{mP}{1+m} \quad \text{— (i)}$$

$$P_c = \frac{P \cdot E}{E_c + E_s} = \frac{P}{1+m} \quad \text{— (ii)}$$

$$m = \frac{E_s}{E_s} \rightarrow \text{modular Ratio}$$

It is seen that the load carried by steel is m times the load carried by concrete

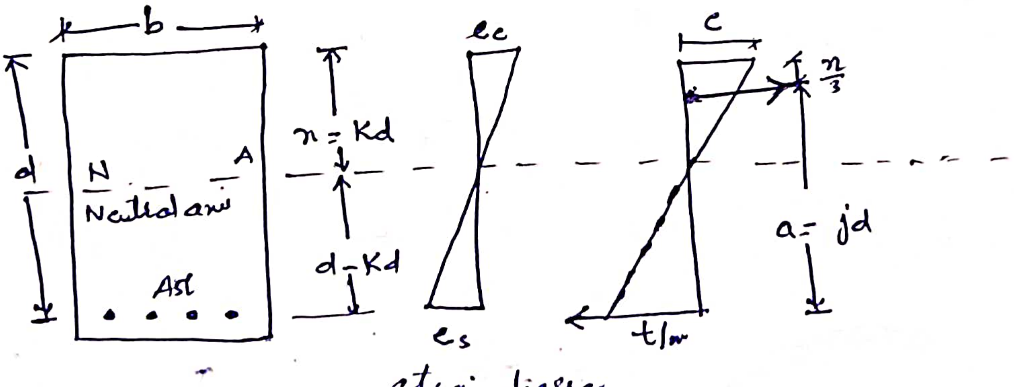
② Equivalent Area of the composite section

$$\boxed{P_s = m P_c}$$

Shows that stress in steel is equal to m times the stress in concrete

$$P_c = \frac{P}{a_c + mA_s} = \frac{P}{A_e}$$

Stresses in concrete and steel



c = compressive stress in the extreme fibre of coarse

t = tensile stress in steel reinforcement

b = Breadth of beam

d = depth of to the centre of reinforcement (effective depth)

n = kd = depth of N.A below the top of the beam

k = neutral axis depth factor = $\frac{n}{d}$

Ast = area of tensile reinforcement

Important formula

① Modular Ratio, $m = \frac{280}{3\sigma_{cbc}} = \frac{E_s}{E_c}$

②. critical neutral axis depth factor:

$$K_c = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$$

③ Percentage of steel,

$$p = \frac{A_{st}}{bd} \times 100$$

$$p = \frac{50mc^2}{t(mc+t)} \quad \text{or} \quad \frac{50\, m\, \sigma_{cbc}^2}{\sigma_{st}(m\sigma_{cbc} + \sigma_{st})}$$

④ Lever arm,

$$j = 1 - \frac{k}{3}$$

⑤ Moment of Resistance, M_r

M_r = force x lever arm

$$M_r = \frac{1}{2}c \cdot kd \cdot b \cdot (jd) = \left(\frac{1}{2}cjk\right) bd^2$$

$$M_r = Rbd^2$$

$$R = \frac{1}{2}cjk \quad \text{or} \quad R = \frac{1}{2}\sigma_{cbc} j k_c$$

$$t\,A_{st}\,jd = \sigma_{st} A_{st}\, jd$$

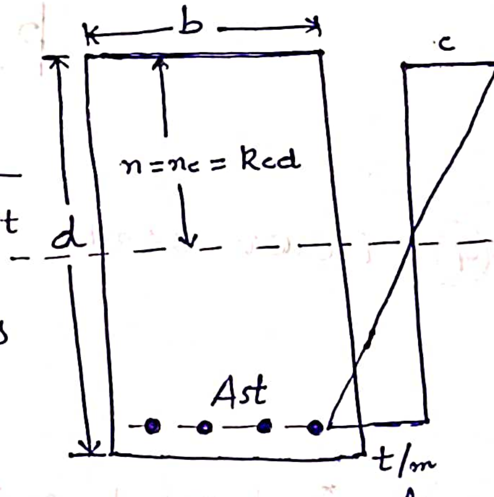
⑥ Area of Steel : Ast

$$Ast = \frac{M_r}{t \cdot jd} = \frac{M}{t \cdot jd} = \frac{M}{\sigma_{st}\, jd}$$

M → External Bending moment

Balanced Section: In a beam section, if the area of steel reinforcement Ast is of such that the permissible stresses $c (= \sigma_{cbc})$ and $t (= \sigma_{st})$ in concrete and steel respectively, are developed simultaneously, the section is known as the balanced section, critical section or economical section.

The neutral axis factor $K_c = \dfrac{m\,\sigma_{cbc}}{m\,\sigma_{cbc} + \sigma_{st}}$



Balanced section

and depth n_c of the critical N.A is

$$n_c = K_c d$$

for Balanced Section

Moment of resistance from compressive force = Moment of resistance from tensile force

$$\left(\frac{1}{2}\sigma_{cbc}\, j K\right) \times bd^2 = \sigma_{st}\, Ast \times jd$$

The % area of steel reinforcement for balanced section

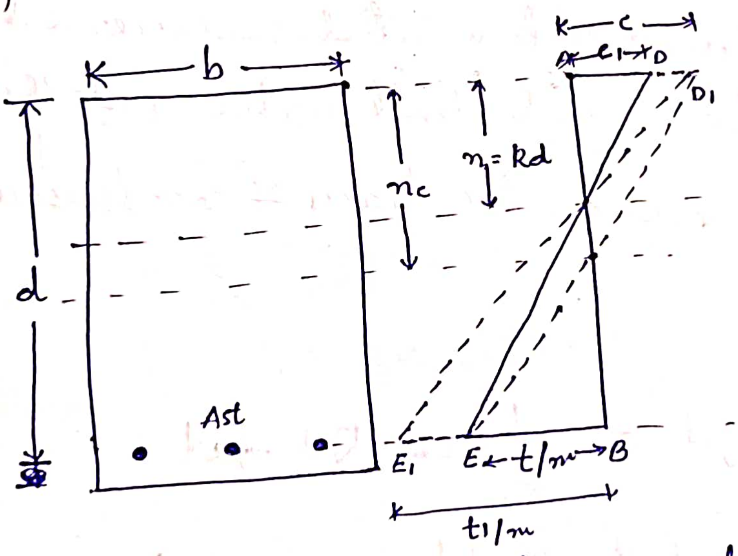
$$p_c = \frac{1}{2} k \cdot \frac{c}{t} \times 100$$

$$p_c = \frac{1}{2} k_c \cdot \frac{\sigma_{cbc}}{\sigma_{st}} \times 100$$

for Fe 250, M15

$$p_c = \frac{1}{2} \times 0.404 \times \frac{5}{140} \times 100 = 0.72\%$$

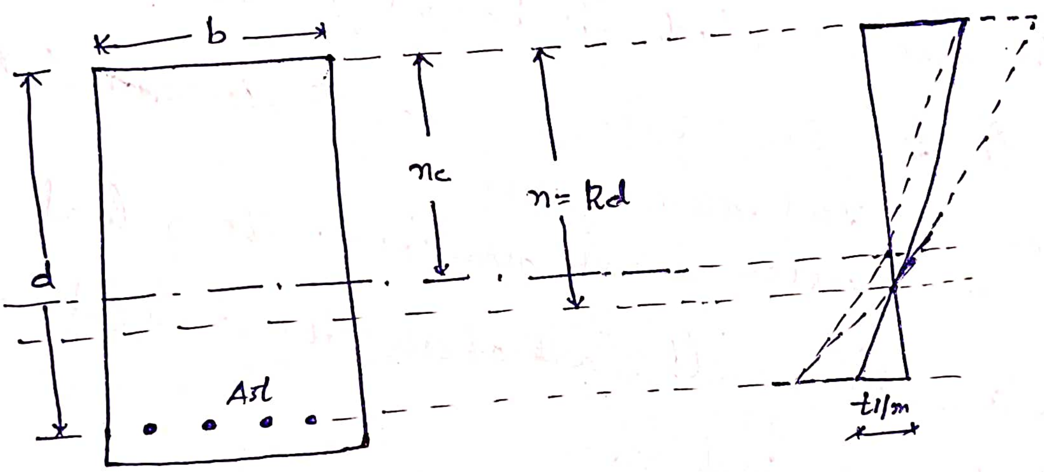
(b) <u>Under - Reinforced Section</u> : An under - reinforced sections is the one in which the percentage steel provides is less that that $p_c = \dfrac{50\, m\, \sigma_{cbc}^2}{\sigma_{st}(m\sigma_{cbc} + \sigma_{st})}$ and therefore full strength of concrete in compression is not developed. the actual neutral axis of such a section will fall above the critical neutral axis of a balanced section.



In this section, the concrete is not fully stressed to its permissible value when the stress in steel reaches its permissible value of $t = \sigma_{st}$. The moment of resistance of an under - reinforced section is, therefore computed on the basis of the tensile force in steel:

$$M_r = t\,A_{st} \cdot jd = \sigma_{st}\, A_{st}\, jd$$

(c) Over - Reinforced Section : In an over reinforced section, the reinforcement provided is more than critical one and therefore the actual N. A of such a section falls below the critical N. A. of a balanced section



Thus, in a over - reinforced section, steel reinforcement is not fully stressed to its permissible limit value and the moment of resistance is determined on the basis of compressive force developed in concrete.

$$M_r = \frac{1}{2} c\, Rd \cdot jd = \frac{1}{2} \sigma_{cbc}\, Rd \cdot jd$$

$$M_r = \frac{1}{2} \sigma_{cbc} \cdot n \left(d - \frac{n}{3}\right) b$$

Determine the moment of resistance of a singly reinforced beam 160mm wide and 300mm deep to the centre of reinforcement, if the stresses in steel and concrete are not to exceed 140N/mm² and 5N/mm². The reinforcement consists of 4 bars of 16 mm diameter. Take, m = 18, If the above beam is used over an effective span 5m, find the ~~the~~ max^m load the beam can carry, inclusive of its own weight.

Sol:

$$A_{st} = 4 \times \frac{\pi}{4} \times (16)^2 = 804 \text{ mm}^2$$

$$c = \sigma_{cbc} = 5 \text{ N/mm}^2, \quad t = \sigma_{st} = 140 \text{ N/mm}^2$$

$$m = 18$$

Equating the moment of area in compression to the moment of area in tension.

i.e $$b \times n \times \frac{n}{2} = m A_{st} (d - n)$$

$$\frac{160 \times n^2}{2} = 18 \times 804 (300 - n)$$

$$n^2 + 181 n - 54281 = 0$$

$$n = \frac{-181 \pm \sqrt{(181)^2 - 4 \times 1 (-54281)}}{2 \times 1}$$

$$n = 159.5 \text{ mm}$$

depth of critical neutral axis

$$n_c = R_c d = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} = \frac{18 \times 5}{18 \times 5 + 140} = 117.4 \text{ mm}$$

Since the actual depth of neutral axis is more than the crit. depth of N.A., the section is over-reinforced. Thus, concrete reaches its max^m stress earlier to steel. Hence the moment of resistance is found on the bases of compressive force developed in concrete

$$\text{Lever arm} = d - \frac{n}{3} = 300 - \frac{159.5}{3}$$

$$= 246.8 \text{ mm}$$

$$M_r = \frac{1}{2} c . n . b . \left(d - \frac{n}{3} \right)$$

$$M_r = \frac{1}{2} \times 5 \times 159.5 \times 160 \, (246.8)$$

$$= 15.75 \times 10^6 \text{ N-mm or } 15.75 \text{ KN-m}$$

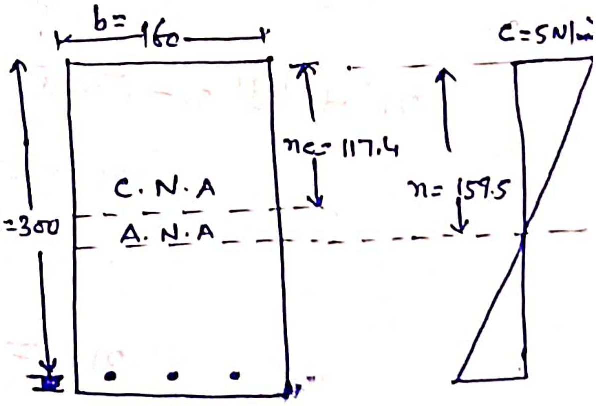
Let, w = uniformly distributed load in KN/m inclusive of the self wt. of the beam

$$M = \text{Max}^m \text{ B.M} = \frac{wl^2}{8} = \frac{w \times (5)^2}{8}$$

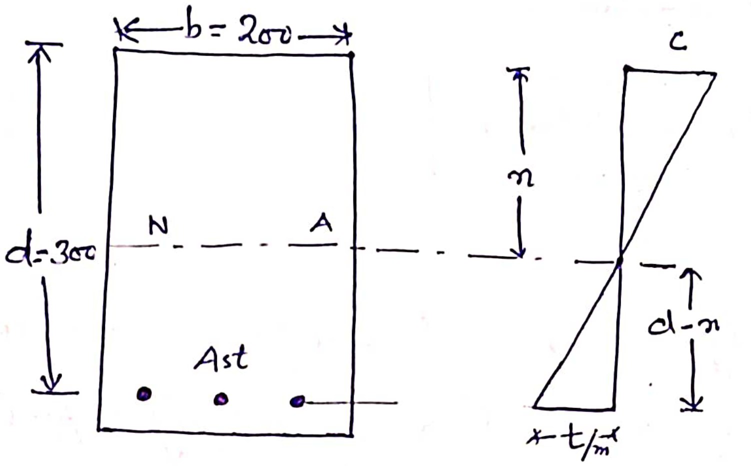
$$M_r = M$$

$$15.75 = \frac{w \times 25}{8}$$

$$w = 5.04 \text{ KN/m}$$



2) The cross section of a simply supported reinforced beam is 200 mm wide and 300 mm deep to the centre of the reinforcement which consists of 3 bars of 16 mm dia. Determine from the first principles the depth of N.A and the max^m. stress in concrete when steel is stressed to 120 N/mm². Take, m = 19



(Sol:) $Ast = 3 \times \frac{\pi}{4}(16)^2 = 603.2\ mm^2$

Let the depth of N.A be n. Equating the moment of compressive area to the moment of equivalent area of steel about N.A.

$$b \cdot n \cdot \frac{n}{2} = m\, Ast\, (d - n)$$

$$\overset{100}{\cancel{200}} \times \frac{n^2}{\cancel{2}} = 19 \times 603.2\,(300 - n)$$

$$n^2 + 114.6n - 34382 = 0$$

$$n = 136.8\ mm$$

Stress in steel, $t = 120\ N/mm^2$

$$\frac{t}{m} = \frac{120}{19} = 6.316$$

from stress diagram

$$\frac{c}{n} = \frac{t/m}{d-n}$$

$$\frac{c}{136.8} = \frac{6.316}{300-136.8}$$

$$\boxed{c = 5.29\ N/mm^2}$$

(Q.3) A rectangular, singly reinforced beam, 300 mm wide and 500 mm effective depth is used as a simply supported beam over an effective span of 6 m. The reinforcement consists of 4 bars of 20 mm diameter. If the beam carries a load of 12 KN/m, inclusive of the self-wt, determine the stresses developed in concrete and steel. Take, m = 19

Sol: Let, n = depth of N.A

$$A_{st} = 4 \times \frac{\pi}{4}(20)^2 = 1256.6\ mm^2$$

Equating the moments of two areas about N.A

$$b \times n \cdot \frac{n}{2} = m A_{st}(d-n)$$

$$300 \times \frac{n^2}{2} = 19 \times 1256.6(500-n)$$

$$n^2 + 159.2n - 79587 = 0$$

$$n = 213.5\ mm$$

lever arm, $a = d - n/3 = 500 - \frac{213.5}{3} = 428.8\ mm$

$$\text{Max}^m \ B.M = \frac{wl^2}{8} = \frac{12 \times (6)^2}{8}$$

$$= 54 \text{ KN-m}$$

$$= 54 \times 10^6 \text{ N-mm}$$

let c be the compressive stress in concrete

$$M_r = \frac{1}{2} c.n.b \times a = \frac{1}{2} c \times 300 \times 213.5 \times 428.8$$

$$= 13.732 \times 10^6 \text{ N-mm (compressive)}$$

Equating M_r to external B.M

$$13.732 \times 10^6 c = 54 \times 10^6$$

$$c = \frac{54}{13.732} = 3.93 \text{ N/mm}^2$$

If t is the corresponding stress in steel

$$t = \frac{mc}{n}(d-n)$$

$$= \frac{19 \times 3.93}{213.5}(500 - 213.5)$$

$$= 100.2 \text{ N/mm}^2 \text{ (Tensile)}$$

(Q.4) Design a reinforced concrete beam subjected to a BM of 20 KN-m. Use M20 concrete, and Fe 415 reinforcement. Keep the width of the beam equal to half the effective depth.

Sol: For M20 concrete, $c = \sigma_{cbc} = 7\ N/mm^2$ and $m = 13.33$

Fe 415 steel, $\sigma_{st} = 230\ N/mm^2$. For a balanced section.

$$k_c = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{13.33 \times 7}{13.33 \times 7 + 230} = 0.289$$

$$j_c = 1 - \frac{k_c}{3} = 1 - \frac{0.289}{3} = 0.904$$

$$R_c = \frac{1}{2} c . j_c . k_c = \frac{1}{2} \times 7 \times 0.904 \times 0.289 = 0.914$$

$$M_r = R_c bd^2 = \frac{R_c d^3}{2} = \frac{0.914 \times d^3}{2} = 0.457 d^3$$

Given, $BM = 20\ KN\text{-}m = 20 \times 10^6\ N\text{-}mm$

$$M_r = BM$$

$$0.457 d^3 = 20 \times 10^6$$

$$d = 352.4\ mm$$

$$b = \frac{1}{2} d = \frac{1}{2} \times 352.4$$

$$b = 176.2\ mm$$

Area of Steel, $A_{st} = \dfrac{M}{\sigma_{st} j_c d} = \dfrac{20 \times 10^6}{230 \times 0.904 \times 352.4}$

$$A_{st} = 273\ mm^2$$

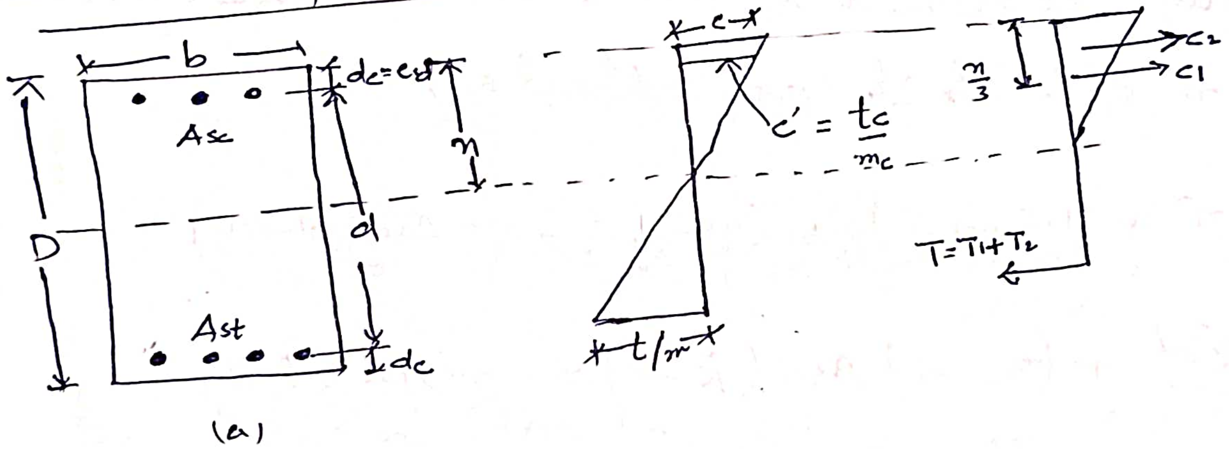
5 (1) find the M.O.R of a R.C.C beam 300mm and 500mm effective depth is required is reinforced with 3 bars of 16mm. use M20 concrete and Fe415 steel. By working stress method.

~~DOBLY~~

Doubly Reinforced Beam

When the dimensions of a beam are restricted by architectural or structural consideration the sections will have insufficient Area of concrete to provide for compressione stresses. In such cases steel is placed in compression and tension zones and such beam are called doubly Reinforced beam.

Location of N.A Axis



(a)

b → breadth of the Beam

d → effective depth of the beam

d_c → depth of the centre of compression steel = e.d.

e → compression steel depth factor = $\frac{d_c}{d}$

c → Max^m stress in concrete

t → " " " steel

c' → stress in concrete surrounding compression steel.

t_c = stress in ~~steel~~ compressive steel $= m_c c' = 1.5 m c'$

Ast → Area of tensile steel

Asc → " " compression "

m_c → modular ratio for compression zone $= 1.5 m$

$$c' = \frac{k-e}{k} \cdot c$$

$$t_c = m_c c' = 1.5 m \frac{k-e}{k} \cdot c$$

$$k = \frac{mc}{mc+t} = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$$

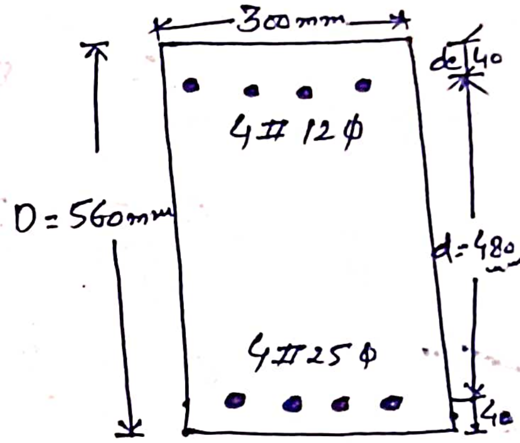
Equating the moment of compression area ~~of the compression~~ about N-A to the m. of the tensile area about N-A

$$\frac{bn^2}{2} + (1.5m - 1) A_{sc} (n - d_c) = m A_{st} (d - n)$$

or $$\frac{bk^2 d}{2} + (m_c - 1) A_{sc} (k - e) = m A_{st} (1 - k)$$

Design a rectangular beam section to carry 16 KN m moment with M20 and Fe415 steel: The overall depth of the beam is restricted to 270 mm.

1) A beam section 300 mm wide and 560 mm deep is reinforced with 4 bars of 25 mm diameter in the tensile zone and 4 bars of 12 mm diameter in the compression zone. The cover to the centre of both the reinforcement is 40 mm. Determine the moment of resistance of the section, if M20 concrete and HYSD bars are used.



Sol:-

$\sigma_{cbc} = 7\ N/mm^2$ [M20 concrete]

$\sigma_{st} = 230\ N/mm^2$ [Fe415]

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$A_{sc} = 4 \times \frac{\pi}{4}(12)^2 = 452.4\ mm^2$$

$$A_{st} = 4 \times \frac{\pi}{4}(25)^2 = 1963.5\ mm^2$$

Position of N.A (neutral axis)

$$\frac{bn^2}{2} + (1.5m - 1)A_{sc}(n - d_c) = m A_{st}(d - n)$$

$$\frac{300 n^2}{2} + (1.5 \times 13.33 - 1)\,452.4\,(n - 40) = 13.33 \times 1963.5\,(480 - n)$$

$$n^2 + 231.8n - 93027 = 0$$

$$n = 210.4\ mm$$

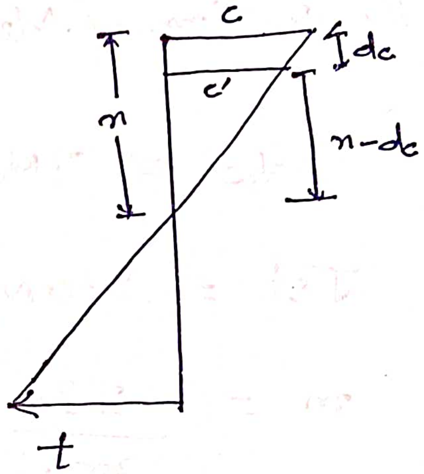
critical N.A, $n_c = \dfrac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}\, d = \dfrac{13.33 \times 7}{13.33 \times 7 + 230} \times 480$

$$n_c = 150.1\ mm$$

Since the actual N.A (n) is below the critical N.A.

$\therefore$ $n > n_c \rightarrow$ the section is over-reinforced section

So, M.O.R ~~is~~ will be found w.r.to concrete



$c = \sigma_{cbc} = 7 N/mm^2$

$$c' = \frac{n - d_c}{n} \times c$$

$$c' = \frac{210.4 - 40}{210.4} \times 7$$

$c' = 5.669 N/mm^2$

$$(M_r)_{con.} = \frac{1}{2} c b n \left(d - \frac{n}{3}\right) + (m_c - 1) A_{sc} \cdot c' (d - d_c)$$

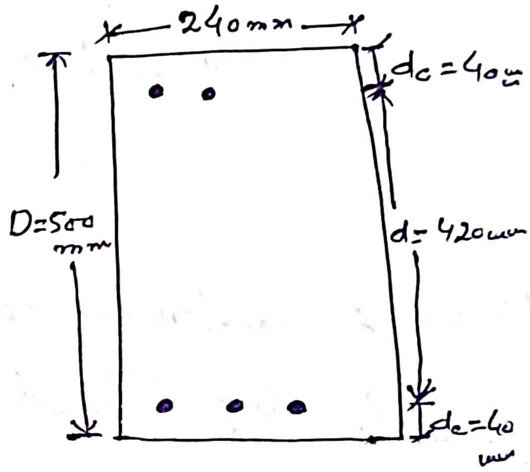
[$m_c = 1.5m$]

$$= \frac{1}{2} \times 7 \times 300 \times 210.4 \left(480 - \frac{210.4}{3}\right) + (1.5 \times 13.33 - 1)\, 452.4 \times 5.669 \times (480 - 40)$$

$(M_r)_{conc.} = 122.77 KN\text{-}m$

(2) A ~~beam~~ Section, ~~300 mm wide~~

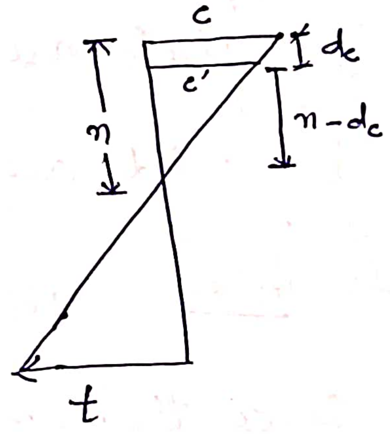
(Q.2) A doubly reinforced rectangular beam is 240 mm wide and 500 mm deep. If the limiting stresses in concrete and steel are 5 N/mm² and 230 N/mm² respectively determine the steel areas for bending moment of 80 KN-m. Assume that steel is buried on both faces with its centre 40 mm from either face. Take m = 19.



Sol:- $c = \sigma_{cbc} = 5\ N/mm^2$, $t = \sigma_{st} = 230\ N/mm^2$

$$n = n_c = \frac{mc}{mc+t}\,d = \frac{19 \times 5}{19 \times 5 + 230} \times 420$$

$$n = n_c = 134.5\ mm$$



$$c' = \frac{n - d_c}{n} \times c$$

$$c' = \frac{134.5 - 40}{134.5} \times 5 = 3.513\ N/mm^2$$

$$M_r = b \cdot n \frac{c}{2}\left(d - \frac{n}{3}\right) + (m_c - 1) A_{sc}\, c'(d - d_c)$$

$[m_c = 1.5m]$

$$80 \times 10^6 = 240 \times 134.5 \times \frac{5}{2}\left(420 - \frac{134.5}{3}\right) + [1.5 \times 19 - 1)\, A_{sc}\, (3.513)[420 - 40]$$

$A_{sc} = 1146\ mm^2$, use 20 mm φ bar

No. of bars $= \frac{A_{sc}}{\frac{\pi}{4} d^2} = \frac{1146}{\frac{\pi}{4}\ldots} =$

Total compression force = Total tension force

$$b n \frac{c}{2} + (m_c - 1) A_{sc} c' = A_{st} \times t$$

$$240 \times \frac{134.5}{2} \times 5 + (1.5 \times 19 - 1)\, 1146 \times 3.513 = A_{st} \times 230$$

$$A_{st} = 832\ mm^2$$

use 16 mm ϕ bars

$$\text{No. of bars} = \frac{A_{st}}{\frac{\pi}{4} d^2} = \frac{832}{\frac{\pi}{4}(16)^2} =$$

(Q.3) A reinforced concrete section is subjected to a reversal of Bending moment of equal magnitude of 120 KN-m in either direction. Design the section, if the permissible stresses in concrete and steel are 5 N/mm² and 140 N/mm² resp. and $m = 15$. Assume $b = 0.6d$ and effective cover to steel equal to $0.1d$.

Sol:- Given: $\sigma_{cbc} = 5\ N/mm^2$, $\sigma_{st} = 140\ N/mm^2$

Since the moment can act in either direction (i.e hogging as well as sagging), we will have to provide equal reinforcement at the top as well as bottom. Hence. $A_{sc} = A_{st} = A_s$.

$$n = n_c = \frac{mc}{mc + t} \times d = \frac{15 \times 5}{15 \times 5 + 140} \times d = 0.391d$$

Equating the moment of area about N.A.

$$\frac{bn^2}{2} + (1.5m - 1) A_{sc}(n - d_c) = m A_{st}(d - n)$$

$$0.6d \times \frac{(0.391d)^2}{2} + (1.5 \times 15 - 1) As (0.391d - 0.1d) = 15 \times As \times (d - 0.391d)$$

or $0.046d^2 + 6.26 As = 9.14 As.$

$$As = Asc = Ast = 0.016 d^2 \ (mm^2)$$

$$M_r = bn \frac{c}{2} \left(d - \frac{n}{3}\right) + (m_c - 1) Asc \cdot c' (d - d_c)$$

$$c' = \frac{n - d_c}{n} \cdot c = \frac{0.391d - 0.1d}{0.391d} \times 6$$

$$c' = \frac{(0.391 - 0.1)\cancel{d}}{0.391 \cancel{d}} \times = 4.46 \ N/mm^2$$

$Asc = 0.016 d^2 \ mm^2$, $M_r = M = 120 \times 10^6$ N-mm

$$120 \times 10^6 = 0.6d \times 0.391d \times \frac{6}{2} \left(d - \frac{0.391d}{3}\right) + (1.5 \times 15 - 1) 0.016d^2 \times 4.46 (d - 0.1d)$$

or $120 \times 10^6 = 0.612d^3 + 1.381d^3 = 1.993d^3.$

$d = 392$ mm

$b = 0.6 \times 392 = 235$ mm

$As = Ast = 0.016 \times (392)^2 = 2460 \ mm^2$

$d_c = 0.1 \times 392 = 40$ mm

use 25mm φ bars

$$\text{No. of bars} = \frac{A_{st}}{\frac{\pi}{4} d^2} = \frac{2460}{\frac{\pi}{4}(25)^2} = 5 \text{ nos.}$$

provide 5 # 25 φ reinforcement

Design a rectangular beam section to carry 16 KN-m moment with M20 and Fe 415 Steel. The overall depth of the beam is restricted to 270 mm.

Sol:->

Limit State method of Design: Limit state design has originated from ultimate or plastic design.

An ideal method is the one which takes into account not only the ultimate strength of the structure but also the Serviceability and durability requirements. Limit state method of design is oriented towards the simultaneous satisfaction of all these requirements. The acceptable limit of Safety and Serviceability requirements, before failure occurs is called a limit state. Two types of limit states are considered in the design.

1. Limit state of collapse.
2. " " " Serviceabilty

1. Limit state of collapse (Safety requirements)

(i) limit state of collapse in flexure (ii) limit state of collapse in compression

(iii) limit state of collapse in Shear (iv) limit state of collapse in torsion.

② Limit State of Serviceability:

(i) Excessive deflection (ii) Premature or excessive cracking

(iii) other limit states (like, vibration, Durability, fire Resistance)

(a) Material Strength : characteristic strength of Materials:

The term chacteristic strength means that value of strength of the material below which not more than 5% percent of the results are expected to fall. It is denoted by fck (N/mm^2)

fck → characteristic strength of concrete

fy → " " " tensile : It is 0.2% of proof strength

(ii) Design Value :- The design strength of the material; Fd is given by

$$F_d = f/\gamma_m$$

f → chacteristic strength of material [fck → concrete, fy → steel]

γ_m → Partial Safety factor appropriate to material.

$\gamma_{mc} \rightarrow 1.5$, $\gamma_{ms} \rightarrow 1.15$

$$f_{yd} \rightarrow \frac{f_y}{\gamma_m} := \frac{f_y}{1.15} = 0.87 f_y$$

$$(f_{ck})_s \rightarrow 0.67 f_{ck}$$

$$(f_{ck})_d \rightarrow \frac{0.67 f_{ck}}{1.5} = 4.446\, f_{ck}$$

Imposed LOADS :

(i) characteristic loads: The term chacteristic load means that value of load which has a 95% ~~probality~~ probability of not being exceeded during the life of the structure.

(ii) Design load : The design load F_d is given by

$$F_d = F(\gamma_f)$$

$F \rightarrow$ chacteristic load, $\gamma_f \rightarrow$ partial Safety factor to loading

(iii) Partial Safety factor : $f(\gamma_f)$

IS - 875 - 1987 Design loads

IS - " " " Part - I $\rightarrow$ Dead load

" " " " " II $\rightarrow$ Imposed "

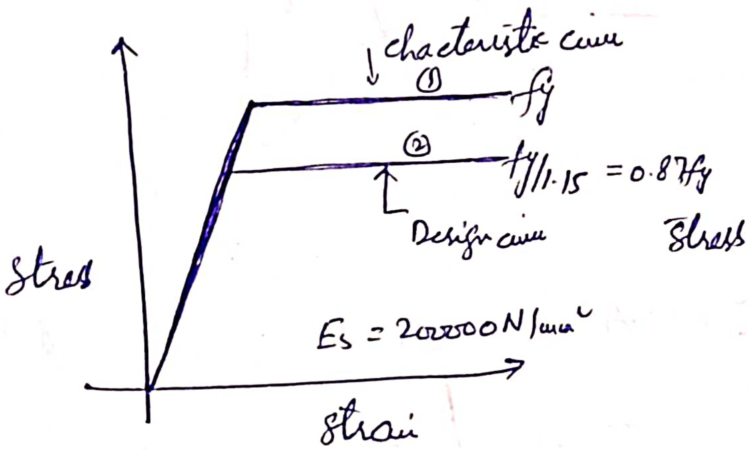
" " " " " III $\rightarrow$ Wind load

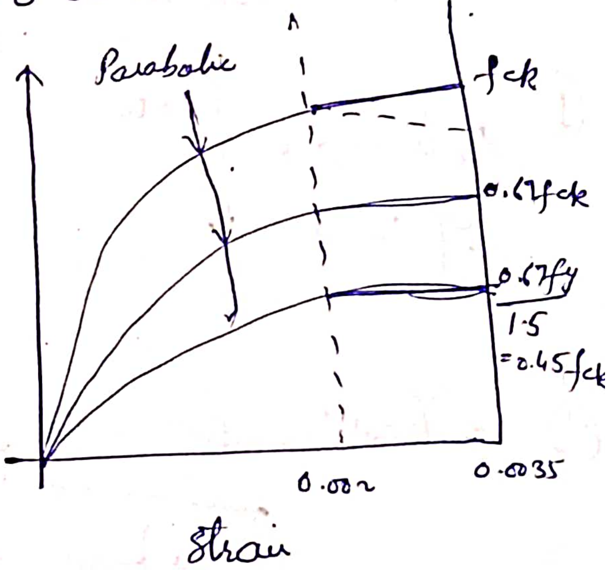
" " " " " IV $\rightarrow$ Snow "

" " " " " V $\rightarrow$ special load and load combinations.

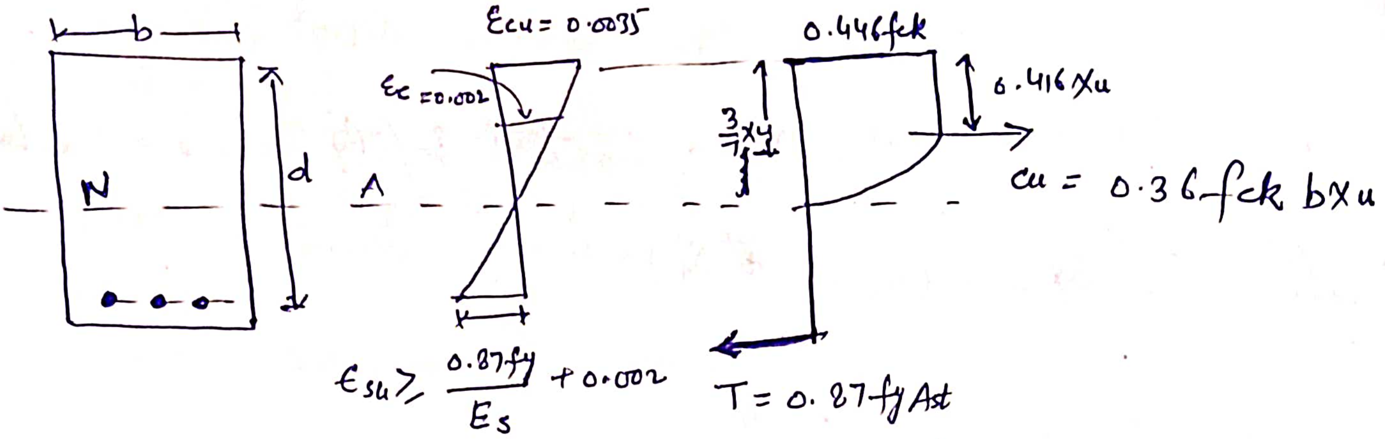
Maxm strain in concrete $\rightarrow$ 0.0035

" " " steel $\rightarrow \frac{f_y}{1.15 E_s} + 0.002$





Design Stress Block Parameter :



Total compression force in concrete $= 0.36 f_{ck} \cdot x_u\, b$

" Tensile force " Steel $= 0.87 f_y A_{st}$

Total compression force = Total tensile force

$$0.36 f_{ck}\, x_u\, b = 0.87 f_y A_{st}$$

① Actual depth of N.A

$$\boxed{x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b}}$$

② critical/limiting depth of N.A

$$\boxed{(x_u)_{max} = \frac{700}{1100 + 0.87 f_y}}$$

fy	250	415	500
$x_{u\,max}$	0.53	0.48	0.46

Moment of resistance (Procedure)

(1) find Depth of N.A

$$\boxed{\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b}}$$

(2) find critical N.A by

$$\boxed{\frac{(x_u)_{max}}{d} = \left(\frac{700}{1100 + 0.87 f_y}\right) d}$$

compare x_u and $(x_u)_{max}$

(a) If $x_u < (x_u)_{max}$, then

(3) M.O.R

$$\boxed{M_u = 0.87 f_y A_{st}\, d\left(1 - \frac{A_{st}}{bd}\frac{f_y}{f_{ck}}\right)}$$

(b) If $x_u \not< (x_u)_{max}$, then

M.O.R, $M_u = 0.36 f_{ck} \frac{(x_u)_{max}}{d}\left(d - 0.416\, \frac{x_{u\,max}}{d}\right) b$

$$\boxed{M_u = 0.36 f_{ck} (x_u)_{max}\, b\, (d - 0.416\, x_{u\,max})}$$

(4) Area of steel :- Ast

$$M_u = 0.87 f_y A_{st} (d - 0.416\, x_u)$$

$$\boxed{A_{st} = \frac{M_u}{0.87 f_y (d - 0.416\, x_u)}}$$

or, $\boxed{A_{st} = \frac{p_t\, bd}{\cancel{bd}}}$

(Q1) Find the M.R of a singly reinforced concrete beam of 200mm width and 400 mm effective depth, reinforced with 4 bars of 16mm dia of Fe415 steel. Take M20 concrete. use I.S code method. Redesign the beam if necessary.

Sol:-

$$A_{st} = 4\times\frac{\pi}{4}(16)^2 = 804.25\,mm^2, \quad p_t = \frac{A_{st}}{bd} = \frac{804.25}{200\times400} = 0.0101$$

Depth of N.A

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b} = \frac{0.87\times415\times804.25}{0.36\times20\times200}$$

$$x_u = 201.64\,mm$$

Limiting depth of N.A

$$(x_u)_{max^m} = 0.48d = 0.48\times400 = 192\,mm$$

$x_u > x_{u\,max^m}$)

Thus, the actual N.A depth is more than the limiting one, such a beam is over-reinforced and hence is undesirable, such a beam should be redesigned.

$$(M_u)_{lim} = 0.36 f_{ck}\, x_{u\,max^m}(d - 0.416\, x_{u\,max^m})\cdot b$$

$$= 0.36\times20\times192(400 - 0.416\times192)\times200$$

$$= 88.5\times10^6\ N\text{-}mm$$

~~% of steel~~ Area of steel: Ast

$$A_{st} = \frac{0.36 f_{ck}\, b\, x_{u\,max^m}}{0.87\times f_y} = \frac{0.36\times20\times200\times192}{0.87\times415} = 765.7\,mm^2$$

2) A rectangular beam 200mm wide and 400mm effective depth is reinforced with 3 bars of 16mm diameter. If grade of concrete is M20 and grade of steel Fe415, determine the bending moment capacity of the Beam.

Sol:- Given:- Effective depth = d = 400mm

width , b = 200mm

$A_{st} = 3 \times \frac{\pi}{4}(16)^2 = 603.2\,mm^2$

To find: Bending moment capacity (i.e M.O.R)

① Depth of N.A

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 603.2}{0.36 \times 20 \times 200}$$

$x_u = 151.24\,mm$

② critical Depth of N.A (x_{umax})

$$(x_{umax}) = 0.48d \quad [Fe415]$$

$= 0.48 \times 400$

$= 192\,mm$

Since $x_{umax} > x_u$, then the section is under reinforced section. Hence, the M.O.R ~~should~~ will be found with respect to ~~concrete~~ steel.

M.O.R

$$M_u = \underline{0.87 f_y A_{st}} \left(d - 0.416 x_u \right)$$

$$= 0.87 \times 415 \times 603.2 \left(400 - 0.416 \times 151.24 \right)$$

$$= 73.41 \times 10^6 \text{ N-mm} \quad \text{or } 73.41 \text{ KN-m}$$

(Q.3) Design a balanced singly reinforced concrete beam section for an applied moment of 60 KN-m. The width of the beam is limited to 175 mm. Use M20 concrete and Fe 415 steel bars.

Sol:- Given: applied Moment, $M = 60$ KN-m

$\gamma_f = 1.5$

$b = 175$ mm

Design Moment, $M_D = 1.5 \times 60 = 90$ KN-m

$= 90 \times 10^6$ N-mm

for Balanced Section

$$x_u = x_{u\,max^m}$$

$$x_{u\,max^m} = 0.48 d \quad [\text{for Fe } 415]$$

$$(x_u)_{max^m} = 0.48 \times d$$

$$(M.O.R)_c = 0.36 f_{ck}\, b\, x_{u\,max^m} \left(d - 0.416\, x_{u\,max^m} \right)$$

$$= 0.36 \times 20 \times 175 \times 0.48 d \left(d - 0.416 \times 0.48 d \right)$$

$$= 484.03 d^2$$

As we know that

$$M_D = (M_r) = 484.03 d^2$$

$$90 \times 10^6 = 484.03 d^2$$

$$d = 431.2 \text{ mm} \simeq 432 \text{ mm}$$

Area of steel: Ast

$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.416 x_{u max})}$$

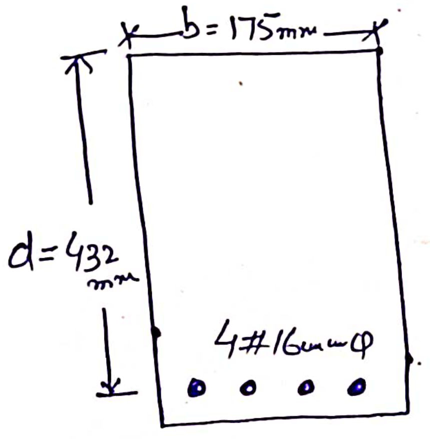
$$A_{st} = \frac{90 \times 10^6}{0.87 \times 415 (432 - 0.416 \times 0.48 \times 432)}$$

$$A_{st} = 720.98 \text{ mm}^2$$

use 16mm ϕ bars

$$\text{No. of bars} = \frac{A_{st}}{\frac{\pi}{4} d^2} = \frac{720.98}{\frac{\pi}{4}(16)^2} = 3.58 \simeq 4 \text{ nos}$$

$\therefore$ provide 4 # 16mm ϕ

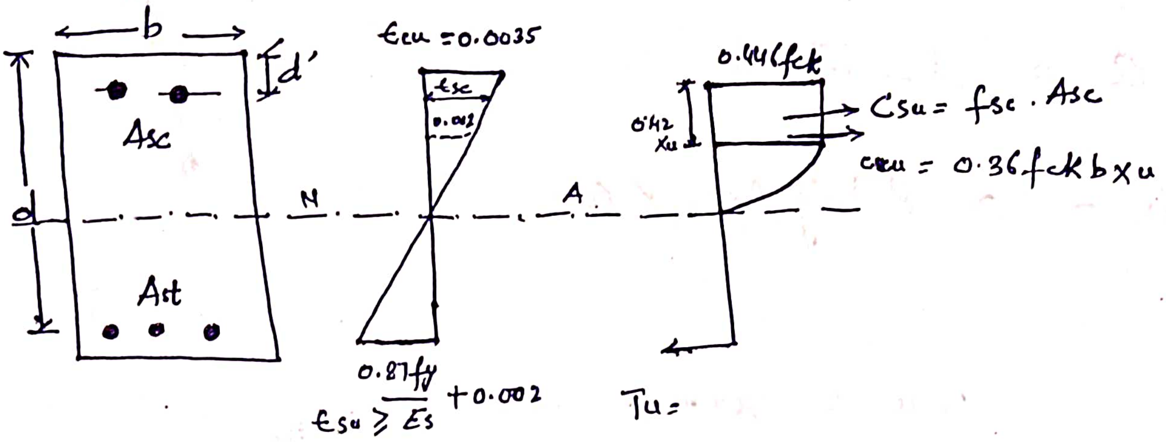


Q4 Find the ~~moment~~ reinforcement for the beam of (Q.3) if the effective depth of the beam is kept equal to 500 mm.

Q.5) A beam, simply supported over an effective span of 7 m carries a live load of 20 KN/m. Design the beam, using M_{20} concrete and HYSD bars of grade Fe415 keep the width equal to half the effective depth, Assume unit weight of concrete as 25 KN/m^3

(Q.6) A reinforced concrete beam has width equal to 300 mm and total depth equal to 700 mm, with a cover of 40 mm to the centre of the reinforcement. Design the beam if it is subjected to a total bending moment of 150 KN-m. use M_{20} concrete and HYSD bars of grade 415

Doubly Reinforced Beams : Beams which are reinforced in both compression and tension sides are called doubly reinforced beams.



Doubly Reinforced Beam

$$M_u = 0.36 f_{ck} x_u b (d - 0.42 x_u) + A_{sc} f_{sc} (d - d')$$

The centroid $\bar{x}$ of the compressive forces from the given eqn

$$\bar{x} = \frac{0.36 f_{ck} b x_u * 0.42 x_u + f_{sc} A_{sc} d'}{0.36 f_{ck} x_u b + f_{sc} A_{sc}}$$

$\therefore$ lever arm, $a = d - \bar{x}$ =

$$(M_u)_s = 0.87 f_y A_{st} \times a$$

$$\boxed{x_u = \frac{0.87 f_y A_{st} - f_{sc} A_{sc}}{0.36 f_{ck} b}}$$

A_{st} → Total reinforcement at tension face, A_{sc} = Reinforcement in compression face

x_u → Depth of N.A, C_{cu} → compressive force in concrete = $0.36 f_{ck} b x_u$

C_{su} → compressive force in compression steel = $f_{sc} A_{sc}$, f_{sc} → Design stress in compression reinforcement

$$\bar{x} = \frac{0.36 f_{ck} b x_u (0.42 x_u) + (f_{sc} - 0.446 f_{ck}) A_{sc} d'}{0.36 f_{ck} x_u b + (f_{sc} - 0.446 f_{ck}) A_{sc}}$$

$$a = d - \bar{x}$$

$$M_u = 0.87 f_y A_{st} \times a$$

<u>Case-2</u>

$$x_u = x_{u\,maxm}$$

$$M_u = 0.36 f_{ck} b x_{u\,maxm} (d - 0.42 x_{u\,maxm}) + A_{sc} f_{sc} (d - d')$$

$$= 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} f_{sc} (d - d')$$

$M_u = 0.36$

<u>Find out M.O.R (Procedure)</u> [given, b, d, Asc, Ast

① Find critical N.A

$$x_{u\,maxm} = \frac{700}{1100 + 0.87 f_y}$$ or

f_y	250	415	500
$x_{u\,max}$	0.46d	0.48d	0.53d

② find the value of f_{sc} [by the table corresponding to steel grade] and strain, ~~ϵ_{scu}~~ ϵ_{sc}

$$\epsilon_{sc} = \left(\frac{x_{u\,maxm} - d'}{x_u}\right) 0.0035$$

③ find the value of ~~x_u~~ Depth of N.A: x_u

$$0.36 f_{ck} b x_u + f_{sc} A_{sc} - 0.466 f_{ck} A_{sc} = 0.87 f_y A_{st}$$

④ find the moment of resistance w.r.to concrete

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} f_{sc} (d - d')$$

(1) find the M.R of an existing beam of M15 concrete, 200 mm wide and 400 mm effective depth, reinforced with 4 bars of (Mild Steel) of 20mm ϕ for tension and 2 bars of 20mm dia for compression. The cover to compression reinforcement is 50mm

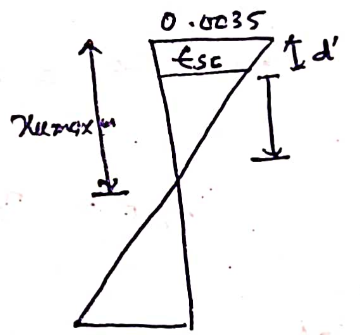
Given: $f_{ck} = 15\,N/mm^2$, $f_y = 250\,N/mm^2$, $b = 200\,mm$, $d = 400\,mm$

$d' = 50\,mm$, $A_{st} = 4 \times \frac{\pi}{4}(20)^2 = 1256.6\,mm^2$, $A_{sc} = 2 \times \frac{\pi}{4}(20)^2 = 628.3\,mm^2$

(1) Critical N.A

$$x_{umax} = 0.46d = 0.46 \times 400 = 184\,mm$$

(2) The value of f_{sc}



$$\varepsilon_{sc} = \frac{(x_{umax} - d')\,0.0035}{x_u}$$

$$\varepsilon_{sc} = \frac{(184 - 50)\,0.0035}{184} = 0.0025 > 0.2\%\,[0.002]$$

Then, $f_{sc} = 0.87 f_y = 0.87 \times 250 = 217.5\,N/mm^2$

(3) ~~find~~ The value of N.A (x_u)

$$0.36 f_{ck} b x_u + f_{sc} A_{sc} - 0.446 f_{ck} A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 15 \times 200\, x_u + 217.5 \times 628.3 - 0.446 \times 15 \times 628.3 = 0.87 \times 250 \times 1256.6$$

$$x_u = 130\,mm$$

$$x_u < x_{umax}$$

Moment of resistance Mu

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 15 \times 200 \times 130 (400 - 0.42 \times 130) + 217.5 \times 628.3 (400 - 50)$$

$$= 96.32 \times 10^6 \text{ N-mm}$$

(Q.2) Determine the ultimate moment of resistance of a doubly reinforced beam section with the following data: $b = 350$mm, $d = 550$mm, $d' = 60$mm

$A_{st} = 5-32$mm ϕ bars $A_{sc} = 3-25$mm ϕ, $f_y = 415$MPa, $f_{ck} = 25$MPa

Given:- $A_{st} = 5 \times \frac{\pi}{4}(32)^2 = 4021.24 \text{mm}^2$

$$A_{sc} = 3 \times \frac{\pi}{4}(25)^2 = 1472.62 \text{mm}^2$$

① critical depth of N.A

$$x_{umax} = 0.48d = 0.48 \times 550 = 264 \text{mm}$$

② strain in compression steel ϵ_{sc}

$$\epsilon_{sc} = \left(\frac{x_{umax} - d'}{x_{umax}}\right) 0.0035$$

$$\epsilon_{sc} = \left(\frac{264 - 60}{264}\right) \times 0.0035$$

$$\epsilon_{sc} = 0.0027$$

$$(f_{sc})_{0.0027} = \frac{351.8 - 342.8}{0.00276 - 0.00241}(0.0027 - .00241) + 342.8$$

$$f_{sc} = 351 \text{ N/mm}^2$$

③ <u>Position of N.A (x_u)</u>:

$$0.36 f_{ck} b x_u + f_{sc} A_{sc} - 0.446 f_{ck} A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 25 \times 350 \times x_u + 351 \times 1472.62 - 0.446 \times 25 \times 1472.62 = 0.87 \times 415 \times 4021.24$$

$$x_u = 302.03 \text{ mm}$$

$x_u > x_{u\,max} \rightarrow$ over reinforced section

④ <u>M.O.R w.r.to concrete</u>

$$M_u = 0.36 f_{ck} b x_{u\,max} (d - 0.42 x_{u\,max}) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 25 \times 350 \times \overset{264}{\cancel{120.03}} (550 - 0.42 \times 264) + 351 \times 1472.62 (550 - 60)$$

$$= 618.45 \text{ KN-m}$$

3) Determine reinforcement of a rectangular beam 300mm wide 400mm effective depth. The beam is subjected to a factored bending moment of 150 KN-m. use M20 concrete and Fe250 steel.

(Q.8) Design a rectangular beam for an effective span 6m. The superimposed load or live load 80 KN/m and the size is limited to 300mm width and 700mm over depth. use M20 concrete mix and Fe 415 steel.

Given:- Overall depth, $D = 700mm$, width of beam, $b = 300mm$

Superimposed load = 80 KN/m, span of beam = $l = 6m$

Assume effective span $d' = 50mm$
" depth = 700 − 50 = 650mm

① Factored Bending Moment

① Moment due to superimposed load, $M = \frac{wl^2}{8} = \frac{80 \times 6^2}{8} = 360 KN\text{-}m$

Dead load of beam $= 0.3 \times 0.7 \times 25 = 5.25 KN/m$

Moment due to dead load $= \frac{5.25 \times 6^2}{8} = 23.625 KN\text{-}m$

Total Moment $= 360 + 23.625 = 383.625 KN\text{-}m$

Factored BM, $M_{uD} = 1.5 \times 383.625 = 575.44 KN\text{-}m$

② limiting Moment of resistance $(M_u)_{lim}$

$(M_u)_{lim} = 0.36 f_{ck} b\, x_{umax} (d - 0.42\, x_{umax})$.

$= 0.36 \times 20 \times 300 \times \overset{312}{\cancel{\underset{226}{650}}} (650 - 0.42 \times \overset{312}{\cancel{\underset{226}{650}}})$

$= 3490.72 \times 10^6 =$

$[x_{umax} = 0.48 \times d = 312mm$
$= 0.48 \times 650 = \cancel{312}$ mm
$=$

Factored BM > $(M_u)_{lim}$ → Hence, the beam is designed as doubly reinforced beam.

③ Area of steel: Ast

$$Ast_1 = \frac{(M_u)_{lim}}{0.87 f_y (d - 0.42 x_{umax})} = \frac{3491.73 \times 10^6}{0.87 \times 415 (650 - 0.42 \times 312)}$$

$$Ast_1 = 1866.52 \, mm^2$$

$$Ast_2 = \frac{M_{uD} - M_{ulim}}{0.87 f_y (d - d')} = \frac{575.44 \times 10^6 - 3491.73 \times 10^6}{0.87 \times 415 (650 - 50)}$$

$$= 1041.91 \, mm^2$$

④ Area of compression steel: Asc

$$Asc = \frac{M_{uD} - M_{ulim}}{f_{sc} (d - d')}$$

$$\varepsilon_{sc} = \frac{0.0035 (x_{umax} - d')}{x_{umax}} = \frac{0.0035 (312 - 50)}{312}$$

$$= 0.00294$$

$f_{sc} =$

$$Ast_2 = \frac{Asc \, f_{sc}}{0.87 f_y}$$